**Butterfly Optimization Algorithm for Model Order Reduction of Linear Section of an Air Core Electrical Transformer**

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**Abstract** In this study, a new evolutionary technique (named as butterfly optimization algorithm (BOA)) is proposed for solution of model order reduction (MOR) problem of a large-scale single-input single-output standard dynamic power system component. The MOR aids investigation and understanding of composite structures with significant properties of the system and input/output characteristic remain unchanged. For comparative performance analysis, the BOA and flower pollination algorithm (FPA) have been simulated for obtaining second and third order reduced model for linear section of an air core electrical transformer. The simulation output results offered in this study indicate capability of the BOA technique which demonstrates its superiority in order to work out the MOR problem in comparison with the FPA and other state-of-the-art evolutionary methods adopted in the recent literature.

**Keywords** Butterfly optimization algorithm, flower pollination algorithm, model order reduction, power system, transfer function, transformer

**1. Introduction**

**1.1 Problem definition**

Computational science or simulation has become an integral part of today’s technological world and carry out consistently for numerous physical, chemical and other processes, to alleviate the work of technical designers and engineers. However, the simulation of complex physical system of higher order leads to complexity of large dimension and high computational time. Therefore, this is handy and occasionally essential to investigate the prospects of existence for various equations of the similar category although of reduced order, which may well satisfactorily replicate the dominant characteristics of the model taken into consideration. Model order reduction (MOR) is one such technique which offers a lower order equivalent system, while retaining the fundamental and vital characteristics of the original higher order system without neglecting stability, transient responses, steady state error and so on.

Generally, interconnected power systems consisting of large transformer are of higher dimension and are very complex in nature. The study and investigation of these models create computational burden. The complexity of these models further increases due to the emergence of uncertainty results which poses great challenges to both technical system analysts and researchers [1]. To resolve this issue, it is desirable to achieve good approximation of these designs to its corresponding reduced order by order reduction approach for effective simulation and analysis.

**1.2 Background perspective**

To fulfill the requirements of huge demand of effortless, reasonable and faster simulation approach of mostly complex and augmented models, different methods of order reduction and their applications have found references in the recent literatures (see [2-5]) and few are addressed in various text books as well (refer [6-9]). In the literature, order reduction techniques have been broadly classified as time domain and frequency domain reduction methods [10].

To date, reduction of the original system, following the MOR methods in time domain, leads to derivation of lower ordered model from the given complex system in the form of transfer function matrix or state model having some state variables. This approach needs the major overview of overall characteristics or eigenvalues and eigenvectors of higher order systems. Some of the methods available with time domain reduction techniques consist of aggregation method [11] and singular perturbation method [12].

Moreover, there are some noteworthy frequency domain reduction methodologiesin the literatures which try to offer good approximation to their lowered order model like continued fraction expansion technique [13], moment matching method [14, 15], Pade approximation technique [16, 17], Routh approximation process [18], Routh-Hurwitz array scheme [19], stability equation means [20], polynomial differentiation technique [21-23], truncation process [24], dominant pole retention technique [25], factor division means [26], Mihailov stability criteria [27], least square method [28] and so on.

In order to improve mapping among the original higher order and inventive reduced order model, various combinational methods were introduced by the researchers using various numerical calculations. In [29], mixed methods have been used for simplification, such as hybrid reduction with differential evolution with simulated annealing [30-31] in addition to stability equation scheme [32] have been implemented to attain denominator for the original model and Lucas factor division technique [33] has been employed to yield the numerator. Another mixed method has been developed by combining the modified pole clustering with factor division technique to attain quality estimation as reported in [34].In[35], Mihailov criterion has been adopted along with factor division method in order to attain and reduce the higher order single-input single-output (SISO) systems.

In recent times, utilization of analogies related to nature or social systems, namely evolutionary methods, comes as most promising field of research for system analysts. These techniques are finding popularity as design tools and problem solvers because of their versatility and capability to optimize and handle the most complex objective functions in the field of research.

In order to attain improved approximation in recent times, some evolutionary optimization techniques such as genetic algorithm (GA) [36-38], particle swarm optimization (PSO) [39-42], bacterial foraging optimization (BFO) [41] and so on have been applied to some test systems to get its reduced model. The frequency response (or alternately, its impulse or step response) intimacy between actual and reduced model is found via evaluating different error indices approximating integral square error (ISE), integral time absolute error (ITAE) and integral absolute error (IAE) etc.

**1.3 Motivation of the present work**

From the past applications of order reduction techniques available in literature, the inference may be drawn that incorporation of bio-inspired algorithms in reduction techniques offers superior performance over other methods to find the global minima or maxima for constrained optimization model. Although, problem to find out relative best solutions within a neighbor solution set and greater amount of computing time are the reasons or the cause of worry for these algorithms to the greatest extent. In the recent time, newly introduced population-based (specifically, butterfly optimization algorithm (BOA) [43]) has been put into practice for numerical calculation and quite a lot for some engineering optimization problems appear to be solved as in [34-40] and has metaphorically performed better to some extent of the further applied techniques such as GA or PSO.

Recent advancement in the field of order reduction for power system applications offers new opportunities for investigation using new nature inspired algorithm like BOA.

**1.4 Contribution of the present work**

In this work, the MOR objectives are realized with minimum computational effort using BOA and Flower pollination algorithm (FPA). Usefulness of the suggested BOA algorithm technique is experienced for order reduction of a large power system structure component (such as linear section of air core transformer (ACT) model). To that end, performances of these two techniques have been compared for different technical specifications such as (a) performance indices criterion of different optimization algorithms, (b) frequency responses of the reduced model with large scale model, (c) step response outputs of the lowered model and the higher one, (e) impulse response outputs for the reduced and the higher model and (d) comparative analysis for responses in terms of rise time, peak time and settling time. The simulation results reveal effectiveness of the BOA algorithm with respect to FPA and other computational intelligence-based techniques surfaced for the past literatures.

**1.5 Layout of the work**

Commencing with organization for present study is arranged as follows. Within Section 2, problem statement of the proposed study has been discussed. Whereas, the test model of linear section ACT is explained in Section 3. In Section 4, the BOA and FPA techniques are briefly described. The simulation outputs offered are illustrated and indicate discussion as in Section 5. Ultimately, in Section 6 conclusions of the present paper are drawn. In the same section scopes of future work are focussed.

**2. Problem formulation**

Mathematical investigation and considerations in relation to optimization and error index for MOR of linear section of an ACT are furnished in the subsequent sub-sections.

**2.1 Statement of the problem**

Within frequency domain approach, the *n-*th order linear time invariant or unchanged higher dimensional model may be depicted by the means of transfer function model as in (1) [40]

 (1)

where

 : numerator polynomial of,

 : denominator polynomial of and

and : scalar constants .

The lowest order transfer function of order ‘’ may be represented as of (2)

 (2)

where

 : numerator polynomial of ,

 : denominator polynomial of and

and : unknown scalar constants .

The significant aspects of the original higher dimensional model can be retained when the objective is to understand the *k-*th () ordered lower dimensional model in the form of (2) from the higher dimensional model of (1). The aim of this work is to match frequency responses of the original and lower order to be closer to get the desired result. The optimization index needs to be formulated in order to get the reduced order system.

**2.2 Response indices**

The integral of square error belonging to the deviations between the basic system and the reduced ordered system have been adopted as error index (EI) in this work to carry out comparative performance analysis of different MOR techniques. The sinusoidal transfer function model for the systems represented as and in (1) and (2), respectively, may be expressed as in (3) and (4), in order.

 (3)

 (4)

Integral square error (ISE) may be formulated as in (5).

 (5)

The integral absolute error (IAE) can be given by (6).

 (6)

The integral time absolute error (ITAE) can be represented by (7).

 (7)

**3. Test system model description**



**Fig. 1.** ACT section

To obtain the reduced order model of an ACT (as depicted in Fig. 1), the BOA technique has been suggested in this study. Due to the nonexistence of magnetic substances, self and mutual inductances be parameters which are reliant on the arithmetical range or position of the winding conductors which does not depend on current. Therefore, to investigate effect for axial and radial twist on distributed limits of transformer winding, an air core section has been adopted as in [45]. This work is carried out by putting forward the air core linear transformer which mainly consists of in shunt, resistance in series and resistance with respect to the ground where is in series with ground capacitance whereas is self-inductance. When the current passes through the coils then the mutual inductances will be produced between the transformer coils. Therefore, the total current passing through the coil can be represented by (8).

 (8)

where

 : inductance current,

 : resistance current and

 : capacitance current.

From (8), we have,

 (9)

where

 : nodal voltage in the transformer,

 : capacitance,

 : conductance and

 : nodal inductance inverse matrices.

Now, differentiating (9) with reference to ‘*t*’, the equation obtained can be given by (10)-(12)

 (10)

 (11)

 (12)

where

u, w : change in voltage and change in current with respect to time and

L,M : Self and mutual inductances, H.

By the product of the equation (12), and making it in the form of state equation as in (13).

 (13)

 (14)

 The state variable from of linear section of ACT can be represented by (15)

 (15)

 (16)

 (17)

 (18)

where

x : state vector

A : state matrix

B : input matrix

The transformer parameters for the 10 sections model are provided in Table 1.

**Table 1** Test Transformer parameters for transformer section

|  |  |  |  |
| --- | --- | --- | --- |
| Parameters | Values | Parameters | Values |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Using the parameters or variables and by using (16)-(18), the tenth order transfer function may be represented by (19)

 (19)

where





**4. Description of algorithms**

**4.1 Basics of FPA algorithm**

FPA [44] usually performs the transfer of pollens within the flowers and is developed through pollination of the flowering of plants. In this context, population is known as flowers and pollinators are called agents or particles. The pollination may take place using two forms, one type is known as biotic form and whereas the other is named as abiotic form. Biotic form add only 10% of pollination in which any pollinators are not necessary followed by self pollination neither local pollination, while the abiotic form donate 90% of the pollination in which pollinators are involved like insects, bats, birds or other animals. In the abiotic form, the searching agents replace its position from one flower to another produces foraging property with the pollinator which keeps changing position to certain flowers than other pollinators where the speed of movement to the several flowers is named as flower constancy. Flowchart of FPA is represented as in Fig. 2 .Furthermore, the various phases concerned are specified below:

**Fig.2.** Flowchart of the FPA algorithm

Yes

No

Is no. of*new sol.*≤ *old sol*.

?

Evaluate new flower constancy

Update

Output the best solution

 Find the current best *g*\*

Is

*Num Gen* < *Max Gen* ?

Discard

Is

new sol> old sol ?

Set the FPA parameters

Initialize the population of flowers

 Evaluate the flower constancy

 Define switch probability

Find the best solution in initial flower

Is

*rand*> *p*

?

Draw state vector L from Levy distribution

Draw from a uniform distribution [0,1]

Global pollination

Local pollination

Yes

No

Yes

No

Yes

No

1. *Initialization phase of algorithm*: In this phase of the algorithm, the population range and parameter are selected which will come to a decision about the amount of self-pollination and cross pollination. The algorithm continues by initializing particular number of population with each one containing a group of variables which are to be optimized.
2. *Estimation of flower constancy*:In this phase of the algorithm, flower constancy of each population is evaluated which will determine how good their variables will minimize the objective function. Based upon the flower constancy, the population is set and the best among them is to be found out.
3. *Determination of pollination phase*: The algorithm continues for the production of new population based on the parameter, which determines whether this is generated by self pollination or cross pollination which is accomplished by producing the random value between 0 and 1 and by comparing with. Now, if the random value is lesser than , then global pollination takes place or it will go for local pollination.
4. *Global pollination:* During this phase, Levy distribution of flight is reproduced in which the flowers will change its position having varying step rangelength from one flower to another one and is given in (20)

  (20)

 where is Levy distribution of flight.

 The new population, which is generated through global pollination, can be represented by (21)

 (21)

where  is the pollen or solution vector at iteration  and  represents the current best solution among all population at current iteration. Here,  is the scaling factor to manage the step size and is the step size parameter that corresponds to the ability of pollination and standard gamma function.

1. *Local pollination:* This pollination takes place within small neighborhood range for the recent population .Therefore, its step range is taken into consideration from a homogeneous distribution which may be, mathematically, represented by (22).

  (22)

1. *Flower constancy is calculated as in Step2:* If the flower constancy of the new population is superior of the earlier population, then they are revised in the location of the earlier one or else, it is detached. The method of generation and comparison processes precede till the count up reaches iteration and current global best is found. This process goes on for the maximum number of iteration and, at last, the current global best is achieved.

The above mentioned steps of the FPA may be described by the flowchart presented in Algorithm 1.

|  |
| --- |
| **Algorithm 1: Computational procedure for FPA algorithm [44]** |
| Objective min or max*dim*=dimensionsInitialize a population of n flowers with random solutions.Find the best solution in the initial population.Define a switch probability.**while** (t < Max Generation) **for** *i*=1:*n* (all *n* flowers in the population) **if** rand <*p*, Draw a step vector L which obeys a Levy distribution. Global pollination using (21). **else** Draw  from a uniform distribution in. Do local pollination using (22). **end if** Evaluate new solutions. If new solutions are better,update them in the population. **end for** Find the current best solution.**end while**Output the best solution found. |

**4.2 BOA algorithm description**

BOA is a newly defined evolutionary algorithm which reflects recent development in the field of research. Butterflies are the key search agents in order to perform optimization in BOA [43]. The butterfly prompts to create fragrance having some intensity that is allied by its fitness value and when butterfly changes its position from one place to the other one, then the fitness value will change consequently. The fragrance will spread over distance so that the other butterflies can sense it which enables the butterflies to share its personal data among others to outline a combined social network. For the global search phase, the butterfly capable to sense fragrance from the others and then it tends to shift towards it whereas in the other condition, for the local phase, the butterfly is not able to sense fragrance from the locality which initiates them to change its position randomly for the BOA [41]. The fragrance is originated as a function of the physical intensity of stimulus as given in (23)

 (23)

where

 : supposed degree of magnitude for the fragrance which shows that how stronger the smell is assumed by other one,

** : tells the sensory modality,

 : stimulus intensity and

: power exponent dependent on modality, which account the variable degree of absorption.

The step by step process of the BOA is summarized in Fig. 3 and the iterations concerned are provided below [43].

Set the BOA parameters

Initialize the population of flowers

Sort the butterflies from individual set and calculate fragrance using (23)

Output the best solution

Is

termination

criterion satisfied

?

Generate random number and switch probability

Is

*rand*> *p*

?

Start local search

using (25)

Start global search

using (24)

Re-evaluate the fitness solution and keep the butterfly having best solution in memory

No

Yes

Yes

No

**Fig. 3.** Flowchart of the BOA algorithm

1. *Initialization phase:* In this step, the technique identifies the benchmark function along with the solution space. The worth or values for the parameters employed for BOA are alloted. The required technique ready to produce an initial population for the butterflies in order to perform optimization after setting the values. As the total number of butterflies keeps unchanged during the simulation of BOA, a fixed size memory is allocated to store their information. The positions of butterflies are randomly generated in the search space, with their fragrance and fitness values are evaluated and stored. This finishes the initialization step and the algorithm goes for the iteration phase, which performs the search with the artificial butterflies created.
2. *Iteration:* The second step of method is known as iteration step in which various numbers of iteration are performed by the algorithm. In every step of the algorithm, all butterflies, in solution space, move to new positions and then, their fitness values are evaluated. The algorithm first calculates the fitness values of all the butterflies on different positions in the solution space. These butterflies will create fragrance at their positions using (23). The iteration step consists of two steps of the algorithm known as global search phase and local search phase.
3. *Global search phase:* In this step, the butterfly moves towards the fittest solution that can be shown by (24)

 (24)

where  represents the solution vector  for *i-*th butterfly in the iteration number ,  shows the current best solution found among all the solutions in the current iteration,represents the fragrance of *i-*th butterfly and random number in [0, 1] is given by .

1. *Local search phase:* In local search phase, local random walk is searched which is given by (25)

in order, (25)

where and  represent, respectively, the *j-*th and *k-*th butterflies from the solution space. Equation (25) turn out to be a local random walk when  along with  belong to the same swarm with is a random number between [0, 1].

The important fraction is an overall mating partner or food searching activities of butterflies. So, the switch probability is used in BOA algorithm to change between common global searches to intensive local search.

1. *Final phase:* In this step, the technique gives the global best solution found with its best fitness value.

According to the above mentioned steps of the BOA, the butterflies will follow three main jobs *viz*., (*a*) all the butterflies are believed to spread the smell that allows the butterflies to pull towards each other; (*b*) each agent changes position randomly and to the suitable agent spreading more fragrance and (*c*) inspiration intensity of a butterfly is affected and resolute with the scenery of the benchmark function.

The computational procedure of the algorithm is summarized in Algorithm 2.

|  |
| --- |
| **Algorithm 2: Computational procedure for BOA algorithm [43]** |
| Objective function*dim*=dimensions.Generate initial population of *n* butterflies.Stimulus intensityat  is determined by.Define sensor modality power exponent and switch probability.**while** stopping criteria not met **do** **for each** butterfly *bf* in population **do** Calculate fragrance for using (23**).** **end for** **Find the best**  **for each butterfly in population do** Generate a random number r from[0, 1]. **if** r <*p* **then** Move towards best solution using (24). **else** Move randomly using (25). **end if** **end for** Update the value of power exponent.**end while**Output the best solution found. |

**5. Test systems vis-à-vis simulation results and discussions**

From the current effort of study, both FPA and BOA are implemented to work out higher order transfer function for the ACT model. The algorithms are employed in MATLAB 2008a computing environment for 2.63 GHz Pentium IV personal computer having 4 GB RAM. The discussions from simulation result outputs on the current effort of work are illustrated in the following sub-sections. Results of interest are **bold faced** in the respective tables to represent the potential optimization ability of the proposed BOA technique.

**5.1. Case 1: Second order reduced model**

The reduced second order transfer function for the implemented test system, given as in (19), has been obtained using FPA and BOA algorithm. The reduced system transfer function for FPA, BOA and other algorithms taken from literature are presented in Table 2 and comparative analysis on various indices is provided in the subsequent sub-sections.

**Table 2.** Second order reduced transfer function offered by various algorithms for standard test system

|  |  |
| --- | --- |
| Algorithms | Transfer function |
| Hybrid Reduction[30] |  |
| DE[31] |  |
| SA[31] |  |
| PSO[41] |  |
| FPA[Studied] |  |
| **BOA[Proposed]** |  |

*5.1.1 Optimization and performance indices comparison*

The optimization and performance indices comparison for the transfer functions given in Table 2 using both FPA and BOA along with other MOR techniques are tabulated in Table3 using (5) - (7)*.* It may be seen from the table that BOA gives a reduction of **6.87%, 19.31%** and **11.42%** in terms of ISE, IAE and ITAE when time taken is 10 seconds as comparable with previous result of PSO [41]. Further, this table illustrates that an improvement of **8.17×**$10^{-4}$ (from previous best of 8.292×$10^{-5}$ (as reported for PSO in [41] to **1.2069×**$10^{-6}$) in the value of EI is accomplished by using the proposed BOA approach.

**Table 3.** Error index of the algorithms for second order

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Algorithms | ISE | IAE | ITAE | Time (sec) |
| Hybrid Reduction [30] | 2.1009×$10^{-9}$ | - | - | - |
| DE [31] | 1.4638×$10^{-6}$ | - | - | - |
| SA [31] | 2.1800×$10^{-6}$ | - | - | - |
| PSO[41] | 8.292×$10^{-5}$ | 3.09×$10^{-4}$ | 2.57×$10^{-5}$ | 10 |
| FPA [Studied] | 3.7618×$10^{-6}$ | 3.74×$10^{-5}$ | 8.27×$10^{-6}$ | 10 |
| **BOA [Proposed]** | **1.2069×**$10^{-6}$ | **1.60×**$10^{-5}$ | **2.25×**$10^{-6}$ | **10** |
| An entry ‘-’ means not available. |

*5.1.2 Frequency response matching*

The comparative Bode plot of the tenth order test system transfer function along with the reduced second order transfer functions given by FPA and BOA are represented as in Fig. 4. From this figure it may be interesting to see that the magnitude as well as phase angle profiles of the proposed BOA based transfer function are approximately matching with the tenth order test system transfer function in the frequency ranges of system operation.

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**Fig. 4.** Bode diagram of reduced model (2ndorder) with the standard model (10th order)

*5.1.3 Step response validation*

The step response of the ACT tenth order system along with the reduced second order system may be observed in Fig. 5. Performance indices of the standard as well as FPA and BOA based reduced order system are represented in Table 4 along with other reduced order systems like hybrid reduction [30], SA [31], DE [31] and PSO [41]. It may be seen from Table 4 that BOA offers perfection in results in terms of rise time, peak time and settling time, respectively, as compared to previous best results offered by PSO based transfer function (as reported in [41]).

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**Fig. 5.** Step responses of reduced model (2ndorder) with the standard model (10th order).

**Table 4.** Comparative analysis of step responses of the standard system with other algorithms for second order

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Algorithms | Order | Peak time(sec) | Settling time(sec) | Rise time(sec) | Correctiongain |
| Standard system | 10th | 2.5826 | 1.2006 | 0.6567 | - |
| Hybrid reduction[30] | 2nd | 2.4915 | 1.2750 | 0.6930 | 0.1570 |
| DE [31] | 2nd | 3.7746 | 1.1913 | 0.6426 | 0.1559 |
| SA[31] | 2nd | 3.7746 | 1.1852 | 0.6369 | 0.1569 |
| PSO[41] | 2nd | 2.2699 | 1.1462 | 0.6389 | 0.1025 |
| FPA [Studied] | 2nd | 2.2697 | 1.1462 | 0.6389 | 0.1025 |
| **BOA [Proposed]** | **2nd**  | **2.2669** |  **1.1448** | **0.6388** | **0.1025** |
| An entry ‘-’ means not available. |

The error between the steady state values of the step responses is found out. This error is expected, because the dc gain is not considered in the optimization index. In order to cancel this error, the correction-gain (CG) is employed while employing (26)

 (26)

where

*DCG* : DC gain of the main system.

The CG of all the algorithms under consideration is tabulated in Table 4. Using this factor, the step response of the reduced system is corrected as illustrated in Fig. 6.

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**Fig. 6.**Step responses of reduced model (2ndorder) with the standard model (10th order) using correction gain.

*5.1.4 Impulse response validation*

The comparative impulse response plot of the tenth order test system transfer function along with the reduced second order transfer functions offered by FPA and BOA are represented as in Fig. 7. From this figure it is observed for the impulse plot profiles is that the proposed BOA based transfer function is almost matching with the tenth order test system transfer function in the time ranges of system operation.

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**Fig. 7**. Impulse responses of reduced model (2nd order) with the standard model (10th order)

**5.2. Case 2: Third order reduced model**

Both FPA and BOA based third order reduced models have been obtained for the tenth order test system transfer function and are presented in Table 5. The numeric results vis-à-vis comparative analysis of the reduced order system performance is presented in the upcoming sub-sections.

**Table 5** Third order reduced transfer function offered by various algorithms for standard test system

|  |  |
| --- | --- |
| Algorithms | Transfer function |
| FPA [Studied] |  |
| **BOA [Proposed]** |  |

*5.2.1 Optimization and performance indices comparison*

FPA as well as the proposed BOA based optimization and performance indices are given in Table 6. It may be seen from the table that BOA offers **0.993%, 1.932%** and **4.92%** in terms of ISE, IAE and ITAE, in order, when time taken is 10 seconds as with FPA algorithm.

**Table 6.** Error index of the algorithms for third order

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Algorithms | ISE(×$10^{-7}$) | IAE(×$10^{-6}$) | ITAE(×$10^{-7}$) | Time (sec) |
| FPA [Studied] | 2.255 | 3.35 | 2.0675 | 10 |
| **BOA [Proposed]** | **1.262** | **1.418** | **1.5752** | **10** |

*5.2.2 Frequency response matching*

Fig. 7 portrays the Bode magnitude and phase angle plots for ACT tenth order system along with reduced third order systems as obtained using FPA and BOA algorithms. It may be noted from the comparative visualization of this figure that the two curves are almost over lapping in the frequency ranges of the test power system operation.

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**Fig. 7**. Impulse responses of reduced model (2nd order) with the standard model (10th order)

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**Fig. 8.** Bode diagram of reduced model (3rdorder) with the standard model (10th order)

*5.2.3 Step response validation*

The time responses of the FPA and BOA algorithms based reduced order system along with the ACT tenth order system for unit step input signal are depicted in Fig. 9. Performance indices for the standard tenth order ACT system and that for the FPA and BOA based reduced third order system are tabulated in Table 7. From this table, an improvement of **9.01%, 4.397%** and **49.86%** in terms of peak time response, settling time response along with rise time response, respectively may be indicated by using the proposed BOA technique as compared to FPA based transfer function.

Further, error between the steady state step responses of the responses is found. This error is expected, because the dc gain is not considered in the optimization index. In order to eliminate this error, the value of CG, defined in (26), is multiplied to the reduced order transfer function. The corrected step response is given in Fig. 10.

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**Fig. 9**.Step responses of reduced model (3rdorder) with the standard model (10th order).

**Table 7.** Comparative analysis of step responses of the standard system with other algorithms for third order

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Algorithms | Order | Peak time(sec) | Settling time(sec) | Rise time (sec) |
| Standard System | 10th | 2.5826 | 1.2006 | 0.6567 |
| FPA [Studied] | 3rd | 2.2147 | 0.5835 | 0.3697 |
| **BOA [Proposed]** | **3rd** | **2.1250** | **0.5465** | 0.3594 |



**Fig.10**.Step responses of reduced model (3rdorder) with the standard model (10th order) using correction gain.

*5.2.4 Impulse response validation*

The comparative impulse response plot of the tenth order test system transfer function along with the reduced third order transfer functions offered by FPA and BOA are represented as in Fig. 11. As of this figure it is notable to observe so as to the impulse plot profiles of the proposed BOA based transfer function is compared with the tenth order test system transfer function in the time ranges of system operation.

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**Fig. 11.** Impulse responses of reduced model (3rd order) with the standard model (10th order).

**6. Conclusions and scope of future work**

In this manuscript, higher order model of ACT section is reduced to its second and third order using newly proposed BOA technique. The frequency response, step response and impulse response information of the higher order dimensional model are well compared as enlisted in its past literature. The proposed method and FPA algorithm have been successfully employed to get its reduced second order and third order form from the higher dimensional tenth order model. The step responses are considered in terms of its rising time and settling time to show effectiveness of the proposed method to reduce its time consumption to find out the solution. Comparative analysis of the response is also presented in terms of performance indices such as ISE, ITAE and IAE. The proposed technique appears to be better in terms of performance indices with respect to other techniques adopted in the past literature. It is concluded that the BOA has the capability to converge towards better quality solution having best convergence characteristics as comparable to previous applied methodologies as reported in the past decades. The proposed reduced order models are found out to be stable and take minimum time to settle down having all the vital characteristics of the original system. Thus, the proposed BOA algorithm may be recommended as a very promising optimizing tool for solution of MOR problem of more complex test power systems or some other more complex engineering optimization problems for the future researchers.

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